Benford’s Law—A Powerful Audit Tool

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The Basics

◆ 1,237 is a number
◆ It is composed of four digits
◆ “1” is the lead digit in the number

Pre-Quiz

In a database of naturally occurring numbers, what are the chances that the lead digit of any given number will be a 1?

a. 10.00% (one out of ten)
b. 11.11% (one out of nine)
c. None of the above
Full Credit for this Presentation Goes to:

*Digital Analysis Using Benford’s Law*, by Mark Nigrini, 2000, Global Audit Publications

**You should buy this book**

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**Who Was Benford?**

- Frank Benford was a physicist working for General Electric in the 1920s
- He noticed that his book of logarithm tables was more worn in the front (covering numbers starting in 1s, 2s, and 3s) than the back (covering numbers starting in 7s, 8s, and 9s)
- Why?
- Frank hypothesized that it was because there *WERE* more numbers starting in 1s, 2s, and 3s than numbers starting in 7s, 8s, and 9s.
Benford’s Law in a Nutshell

◆ A Digital Analysis Technique
◆ The first digits of naturally-occurring numbers are not randomly distributed
◆ Distribution of first four digits:
  – 1 -- 30.1%
  – 2 -- 17.6%
  – 3 -- 12.5%
  – 4 -- 9.7%

Intuition

<table>
<thead>
<tr>
<th>First Digit</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1111%</td>
</tr>
<tr>
<td>2</td>
<td>0.1760%</td>
</tr>
<tr>
<td>3</td>
<td>0.1250%</td>
</tr>
<tr>
<td>4</td>
<td>0.0970%</td>
</tr>
<tr>
<td>5</td>
<td>0.0790%</td>
</tr>
<tr>
<td>6</td>
<td>0.0660%</td>
</tr>
<tr>
<td>7</td>
<td>0.0570%</td>
</tr>
<tr>
<td>8</td>
<td>0.0510%</td>
</tr>
<tr>
<td>9</td>
<td>0.0460%</td>
</tr>
</tbody>
</table>

11.11% (1/9)
Mark Nigrini’s Explanation

- **Population data conforms closely to** Benford’s Law
- Take a county with a population of 10,000 that grows at a rate of 5% per year
- It takes:
  - 15 years to grow from 10,000 to 20,000
  - 8 years to grow from 20,000 to 30,000
  - 6 years to grow from 30,000 to 40,000
  - …
  - 2 years to grow from 90,000 to 100,000
- **Naturally-occurring numbers exhibit this same geometric (logarithmic) sequence characteristic**
The Chambered Nautilus exhibits a logarithmic spiral

Benford’s Law

◆ So what?
◆ Use Benford’s Law to identify anomalous transactions in:
  – Accounts payable, expenses, disbursements
  – Accounts receivable and sales
  – Refunds
  – Payroll
  – Estimations in the general ledger
Purchase Order Amounts

Bank Deposit Amounts
Benford’s Law

- Invented, contrived, or manipulated numbers do not follow Benford’s Law
- Doesn’t apply to all data sets
  - sets of data with numbers that have imposed ceilings or floors (IRA contributions or hourly wage rates)
  - Assigned numbers
- Once again, however, human analysis of anomalies is needed

Quiz: Would the following conform to Benford’s Law? *

1) Populations of towns and cities in Europe
2) Number of shares of stock bought/sold by individual companies on the NYSE
3) Same as 2) but on the Hong Kong Stock Exchange
4) Zip codes in the United States
5) Claims paid by an insurance company that imposes a $500 deductible

* Digital Analysis Using Benford’s Law, by Mark Nigrini, 2000, Global Audit Publications
Quiz: Would the following conform to Benford’s Law?

6) Points scored by each NFL team during a season
7) Street numbers for all US homes that have street numbers
8) Invoice-by-invoice disbursement amounts by General Electric during a year
9) Dollar amounts of each American Airlines ticket sold or refunded during a year
10) Height in meters of 10-year-old boys

Quiz: Would the following conform to Benford’s Law?

11) DirecTV invoiced amounts
12) Dollar and cents selling prices for every item in the Giant Food Stores in Ohio
13) Individual sales amounts for new cars in a Mercedes Benz dealership
14) Extended inventory values (quantity x cost) of every item in a Wal-Mart store
15) Populations of the 50 states in the US
A digital analysis success story

- Performance audit of Port of Seattle Construction Management
- Port policy allowed sole source contracting up to $50,000; required limited (3 bids) competition for contracts from $51,000 to $200,000; and required full and open competition for contracts above $200,000
- Any contract could be amended upward by $30,000 without reprocurement
- The Port did not use independent procurement officials; but allowed engineers and project managers to procure and award contracts

Audit Hypothesis and Testing

- These policies created control weaknesses—opportunities—whereby competition requirements could be circumvented and contracts could be awarded based on other than merit or best value
- We used IDEA to perform a Benford’s Law analysis on the database of several thousand consulting contract award amounts
Benford’s Law First Digit Test

```
<table>
<thead>
<tr>
<th>Digit Sequence</th>
<th>Actual Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>660</td>
</tr>
<tr>
<td>2</td>
<td>560</td>
</tr>
<tr>
<td>3</td>
<td>460</td>
</tr>
<tr>
<td>4</td>
<td>360</td>
</tr>
<tr>
<td>5</td>
<td>260</td>
</tr>
<tr>
<td>6</td>
<td>160</td>
</tr>
<tr>
<td>7</td>
<td>110</td>
</tr>
<tr>
<td>8</td>
<td>90</td>
</tr>
<tr>
<td>9</td>
<td>50</td>
</tr>
</tbody>
</table>
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Benford’s Law First Two Digits Test

```
<table>
<thead>
<tr>
<th>Digit Sequence</th>
<th>Actual Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>220</td>
</tr>
<tr>
<td>29</td>
<td>120</td>
</tr>
<tr>
<td>39</td>
<td>60</td>
</tr>
<tr>
<td>49</td>
<td>30</td>
</tr>
<tr>
<td>59</td>
<td>20</td>
</tr>
<tr>
<td>69</td>
<td>10</td>
</tr>
<tr>
<td>79</td>
<td>8</td>
</tr>
<tr>
<td>89</td>
<td>6</td>
</tr>
<tr>
<td>99</td>
<td>4</td>
</tr>
</tbody>
</table>
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Benford’s Law and Circumvention of Competition Requirements

- Port Response: “50K and 200K also coincides with common delegations of authority and therefore contributes to the large amounts contracted within that limitation.”

- Our report cited details of 19 specific examples where the circumvention of competition was clear.

Benford’s Law and Circumvention of Competition Requirements

- More detailed analysis of specific procurements revealed:
  - Instances where consultants were awarded—sole source—multiple sequential $50,000 contracts
  - Instances in which $50,000 contracts were awarded on a sole source basis and then amended to higher amounts without competition
  - Instances in which simultaneous sole source $50,000 contracts were awarded to the same contractor for the same services
Benford’s Law and Circumvention of Competition Requirements

- More detailed analysis of specific procurements revealed:
  - One consulting company received 44 sole source contracts in a 24-month period for $1,178,440
  - One consulting company received 43 sole source contracts in a 24-month period for $1,530,648
  - A contract was awarded (using limited competition) for $75,272 and then amended upward to $5,846,112

Official Reaction to the Audit

- Seattle Post-Intelligencer
  January 7, 2008
  Feds open criminal inquiry into port

- The Seattle Times
  State audit slams port’s waste
  Report: $97.2 million down the drain
  January 7, 2008
  Justice Department to investigate Port

- The Seattle Times
  January 8, 2008
  Possible fraud at Port focus of criminal probe
Port Commission Investigation

Report of:
THE SPECIAL INVESTIGATIVE TEAM
DECEMBER 3, 2008

While the investigatory team did not identify any embezzlement or personal gain, it found ten incidents of fraud in port contracting practices and exposed a culture that tolerated suppressing information from the elected commission.

Benford’s Law is just one of several digital analysis techniques

- **Number Duplication**: digital analysis studies a database and calculates the Number Frequency Factor (NFF).
- The NFF can range from zero (all duplicates) to 1 (no duplicates)
- Fraudsters tend to use more duplicates than would occur naturally.
- Reasons for inordinate duplicate frequencies need to be analyzed.
- E.g. calculate the NFF for each vendor receiving payments.
Benford’s Law is just one of several digital analysis techniques

- **Same Same Different:**
  - Company A has nine accounts payable processing centers around the country.
  - Each center had controls in place to detect/prevent duplicate payments to any vendor.
  - Digital analysis was performed on ALL payments and instances are reported where the *Same* payment is made to the *Same* vendor by *Different* processing centers.

- **Round Numbers:** digital analysis predicts the frequency of occurrence of numbers.
  - Fraudsters tend to over-use round numbers.
  - An inordinate occurrence of round numbers could indicate use of estimates in lieu of actual amounts.
  - E.g. An inordinate occurrence of round numbers in change orders in a construction contract could mean that the project manager is not putting effort into careful and itemized cost analysis.
Benford’s Law is just one of several digital analysis techniques

- **Relative Size Factor**: examines a subset (e.g. payments to vendors) and computes the RSF for each vendor.
- RSF equals the highest payment to the vendor divided by the next to highest payment.
- Vendors with high RSFs are examined in more detail.

Benford’s Law is just one of several digital analysis techniques

- **Last Two Digits**: Not useful for disbursements; but useful in special situations.
- L2D used to find invented or rounded numbers.
- There are 100 last two digit possibilities (00 to 99).
- The test assumes that each combination has a .01 frequency of occurrence.
- Last two digit combinations occurring more (or less) frequently are examined further.
- E.g. fraudsters are unlikely to invent numbers ending in 00 for fear that these numbers look too rounded. Hence, a low frequency of 00 last two digits could indicate fraudulent data.
Benford’s Law is just one of several digital analysis techniques

◆ **My Law**: if initial analysis shows that data do not conform to Benford’s Law, use My Law.
◆ E.g. An insurance company determines that its “claims paid” database does not conform to Benford’s Law.
◆ The company’s actual claims paid digital frequency for a prior period is its “My Law” data.
◆ Current and future data are then compared to the My Law data; and anomalies are studied further.

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Nigrini’s Fraud Examples

◆ **State of Arizona**: Wayne James Nelson created a fake vendor and had 23 checks issued for phony services totaling nearly $2 million.
◆ 90% of his phony invoice amounts started with 7, 8, or 9.
◆ Nelson inadvertently repeated some digits and digit combinations.
Nigrini’s Fraud Examples

- **Fast Food Restaurant**: Owner created false daily sales tapes at the end of each week in order to under-report sales, and thereby evade taxes.
- The false daily sales data deviated drastically from Benford’s Law.

Nigrini’s Fraud Examples

- **Hotel Chain Healthcare Costs**: The company was self-insured and employees submitted claims to the head office. Claims were assessed and approved for payment by a “trusted supervisor.”
- A Benford’s Law First Two Digits test showed a spike for “65.”
- All “65” claims were audited.
- The “trusted supervisor” had paid 13 fraudulent claims (which ended up in her hands) ranging from $6,500 to $6,599 for heart surgery.
- She knew that this was the amount generally paid for minor heart surgery; and she used supposed employees from hotels that had a high proportion of older employees
Nigrini’s Fraud Examples

◆ **Housing Authority**: A housing authority used off-duty police officers to patrol its projects.
◆ The head of security embezzled $500,000 over a 10-year period by creating false time sheets for actual police officers and then cashing the checks “on behalf of” the officers.
◆ A First Two Digits test revealed significant deviation from the Benford’s Law prediction.

Nigrini’s Fraud Examples

◆ **Utility Company**: Performed a First Two Digits test on invoices paid during a year.
◆ Spikes appeared for “20,” “25,” “40,” and “50.”
◆ Auditors selected 14 vendors for further review based on these results.
◆ Valid reasons were found for 13 of 14 vendors.
◆ The last vendor supplied car batteries for the utility’s fleet of vehicles.
◆ The number of batteries invoiced during the year was about twice the number of vehicles.
What happens if fraudsters read Nigrini’s book (or attend this seminar)?

- “these techniques are quite complex and most fraudsters are not rocket scientists.”
- Fraudsters do not see the complete picture; so they would have to create a fraudulent data set that conforms to Benford’s Law—i.e. numerous transactions.
- Fraudsters trying to match Benford’s Law run the risk that the actual data set does not conform.
- “the problem that every fraudster faces is that he or she isn’t just trying to outwit one aspect of one law … but reality in general.”

- BUT, to be safe, do not share this presentation with any of your fraud perp friends or acquaintances.

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